

# UNDERSTANDING ALGORITHMS

SAPTARSHI NASKAR  
TANMOY BISWAS



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# Understanding Algorithms

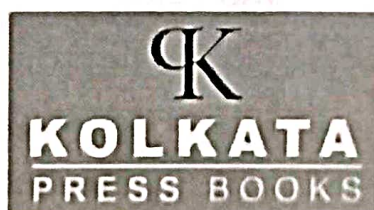
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**FIRST EDITION, FEBRUARY 2022**

DESIGN: TANIA KOLEY

**ISBN: 978-81-955398-2-6**

*Publisher*

**KOLKATA PRESS BOOKS**

CE-322, CHANDIBERIA, SHARODA POLLI,

KRISHNAPUR, KOLKATA-700102

**PH: 9681708009 / 8777019179**

**kolkatapressbooks@gmail.com**

**MRP: ₹300/-**

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## Acknowledgement

I am grateful to my parents *Late Santipda Naskar*, *Late Binapani Naskar*, my inspiration *Sw. Bhudebananda Ji Maharj*, first guru *Sri. Pronab Kumar Ray*, Gurudev *Prof. Samar Sen Samra*, *Prof. Tapas K. Ballav*, elder brother *Dr. Satrajit Ghosh* my beloved friend (*Dr. Krishnendu Basuli*) and my parents-in-laws, brother (*Saptak Naskar*), sisters-in-law *Sayanti Roy*, *Satabdi Hader* and family members whose blessing inspired me to write this book.

The Author would like to thank to his students and colleagues, co-author Tanmoy Biswas who have critically read various portion of the manuscript and offered helpful suggestions. Special thanks go to my wife *Smt. Saswati Roy* my daughter *Ahana Naskar* for their constant support in this work.

I am dedicating my work to my father and mother who had left for heaven and it was their dream hence I dedicate my work to their memory.

*Saptarshi Naskar*

I am very grateful to doctoral guide *Dr. Pranab Roy*, *Dr. Krishnendu Basuli*, *Dr. Diptiman Roy Chowdhury*, *Dr. Rajat Pandit*, *Dr. Amlan Chakraborty*, and all my teachers, whose blessings have inspired me to co-author this book with *Saptarshi Naskar*.

I express sincere gratitude to my colleagues for checking various details and suggesting critical changes for the improvement and development of this book.

I would also like to express my gratitude to my parents *Bijan Bihari Biswas*, *Uttara Biswas*, my brother *Rupayan Biswas*, Sister in law *Sonali Das*, my parents-in-laws, relatives, students for always staying by my side inspiring and encouraging me for such endeavours.

I would specially thank my daughter *Annita Biswas* and my wife *Piyali Mondal* for giving me constant support without which it would have been impossible for me complete this book.

I would also like to acknowledge the support and valuable suggestions received from my beloved friends *Manas Pal* and *Bibek Ranjan Gupta*.

I am dedicating my work to *Aditi* and *my twin babies* who left very early from my life for the heavenly adobe. It was always her dream, and thus I am dedicating this work in their memory.

*We also acknowledge the tireless efforts and consistent support received from the Kolkata Press books team.*

*Tanmoy Biswas*



## Preface

The area of Computer Algorithms started affluent in the early '60s when the users of Electronic Computers started to pay interest to the recital of programs. The limited resources of computers at that time influenced additional momentum for devising efficient Computer Algorithms. After widespread research in this field, numerous efficient Algorithms for different problems emerged. The similarities among different algorithms for certain classes of problems have resulted in general algorithm design techniques. This book emphasizes most of these algorithm design techniques that have proved their effectiveness in the solution to many problems. It may be considered as an attempt to cover the most common techniques in the design and thinking of sequential algorithms.

Although the main initiative of this book is to simplify algorithm design techniques, to analyze the algorithms. Chapter-2 and 3 covers most of the mathematical tools and essential preliminaries chapter-7 covers problem classification and techniques of thinking of algorithms. Chapter-8 convoluted how to think of an algorithm and logic of solution to a problem. These chapters are indispensable for the design of efficient algorithms.

The focus of the presentation is on practical applications of the design techniques. The prerequisites for this book have been kept to nominal; only the elementary background in Discrete Mathematics and Data Structures are implicit.

## Introduction to Algorithms and its applications

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### 1.1 Introduction

**Algorithm** is a procedure that consists of a finite set of instructions which given an input from some set of possible inputs, enables us to obtain an output if such an output exists or else obtain nothing at all if there is no output for that particular input through a systematic execution of some instructions.

When writing a program to be executed in a computer we are generally implementing a method that has been devised previously to solve some problem. This method is most of time independent of the particular programming language being used. It is the method, rather than the computer program itself, that specifies the steps that is used to solve the problem. The term algorithm is used to describe a deterministic, finite, and effective problem-solving method suitable for implementation as a computer program.

The set of possible inputs consists for a particular input set to which the algorithm gives an output. If there is an output for a particular input, then we say that the algorithm can be applied to this inputs and process to give corresponding output.

The design and analysis of algorithms are of fundamental importance in the field of Computer Science.

As Donald E. Knuth states "*Computer Science is the study of Algorithms*". Every area of Computer Science depends heavily on the design of efficient algorithms. Compilers and operating systems are nothing but direct implementations of special purpose algorithms.

The question of whether a problem can be solved using an effusive procedure, which is equivalent to the contemporary notion of the algorithm.

### 1.2 Asymptotic Complexity Analysis of Algorithms

It is meaningless to say that an algorithm A, when presented with input x, runs in time y seconds. This is because the actual time is not only a function of the algorithm used; it is a function of numerous factors, e.g. how and on what kind of machine the algorithm is implemented and in what language and even what compiler or programmer's skills to mention a few. Therefore, we should be content with only an approximation of the exact time. This belongs to an important area in the theory of computation, namely computational complexity. The main objects of study in the field of computational complexity include the time complexity and the space complexity.

(Space complexity: Space needs by an algorithm in order to deliver its output when presented with legal input.)

### 1.3 Order and Growth

When analyzing the running time of an algorithm with another algorithm that solves the same problem, or even a different problem. Thus, our estimate of times is relative as opposed to absolute. It is desirable for an algorithm to be not only machine independent but also capable of being expressed in any language, including human languages. Moreover, it should be technology independent, but also the algorithm to survive technological advances. Our main concern is not in small input sizes, we are mostly concerned with the behaviour of the algorithm under large input instances.

The following operations on fixed-sized operands are examples of elementary operations.

- Arithmetic operations: addition, subtraction, multiplication and division
- Comparisons and logical operations
- Assignments including assignments of pointers (When say, traversing a list of a graph)